Supersymmetric BRST cohomology

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arXiv:1201.3638

Context / applications:

- Quantum field theory: classification of candidate counterterms and anomalies; "algebraic renormalization"
- Classical field theory: classification of conservation laws and consistent deformations

Outline:

- Supersymmetry algebra cohomology (SAC)
- Descent equations
- Elimination of trivial pairs
- Emergence of SAC in local BRST cohomology
- SUSY ladder equations
- Example in D=2
- Comparision to Lie algebra cohomology

Supersymmetry algebra cohomology (SAC)

Supersymmetry (SUSY) algebra, represented on "generalized tensors" $\tilde{T}$:

$$[\,P_a\,,\,P_b\,]=0,\quad [\,P_a\,,\,Q^i_{\underline{\alpha}}\,]=0,\quad \{Q^i_{\underline{\alpha}}\,,\,Q^j_{\underline{\beta}}\}=M^{ij}\,(\Gamma^a C^{-1})_{\underline{\alpha}\underline{\beta}}P_a$$

SUSY differential:

$$s_{\mathsf{susy}}=c^a P_a+\xi^{\underline{\alpha}}_i Q^i_{\underline{\alpha}}-\tfrac{1}{2}\,M^{ij}\,(\Gamma^a C^{-1})_{\underline{\alpha}\underline{\beta}}\,\xi^{\underline{\alpha}}_i\xi^{\underline{\beta}}_j\,\frac{\partial}{\partial c^a}\,,\qquad (s_{\mathsf{susy}})^2=0$$

c^a: anticommuting translation ghosts, $a=1,\dots,D$

$\xi^{\underline{\alpha}}_i$: commuting SUSY ghosts, $\underline{\alpha}=\underline{1},\dots,\underline{2^{\lfloor D/2\rfloor}}$, $i=1,\dots,N$

SAC:

$$H(s_{\mathsf{susy}})=\frac{\text{kernel of } s_{\mathsf{susy}} \text{ in } \Omega}{\text{image of } s_{\mathsf{susy}} \text{ in } \Omega}$$

Representatives of SAC:

$$s_{\mathsf{susy}}\,\omega=0,\quad \omega\sim\omega+s_{\mathsf{susy}}\,\eta,\quad \omega,\eta\in\Omega$$

Ω: space of polynomials $\omega(c,\xi,\tilde{T})$ or similar

Descent equations

Local BRST-cohomology $H(s|d)$, descent equations:

$$\left.\begin{array}{rcl} s\,\omega_D + d\,\omega_{D-1} & = & 0 \\ s\,\omega_{D-1} + d\,\omega_{D-2} & = & 0 \\ & \vdots & \\ s\,\omega_m & = & 0 \end{array}\right\} \Leftrightarrow\ (s+d)\,\tilde{\omega} = 0,\ \tilde{\omega} = \sum_{p=m}^{D} \omega_p$$

In general: descent is trivial, ascent is generally obstructed

$\Rightarrow$ generally, the relation of $H(s)$ and $H(s|d)$ is subtle, $H(s)$ and $H(s+d)$ differ substantially.

In standard supersymmetric field theories (when s contains SUSY): the relation between $H(s)$, $H(s|d)$ and $H(s+d)$ is direct. In particular:

$$s\,\omega = 0\ \Rightarrow\ (s+d)\,\tilde{\omega} = 0 \text{ with } \tilde{\omega} = \omega(c^a \to c^a + dx^a)$$

Reason: $s = c^a\partial_a + \ldots$, $(s+d) = (c^a + dx^a)\,\partial_a + \ldots$

$\Rightarrow$ $H(s)$ contains all relevant information on $H(s|d)$.

$H(s_{\text{susy}})$ contributes to $H(s)$ and thus also directly to $H(s|d)$.

Elimination of trivial pairs

Useful method to analyse the local BRST cohomology: construction of variables u^ℓ, v^ℓ, w^I s.t.

$$(s+d)\,u^\ell = v^\ell \;(\Rightarrow\; (s+d)\,v^\ell = 0), \quad (s+d)\,w^I = r^I(w)$$

This implies, with $\mathcal{F} = \{f(u,v,w)\}$, $\mathcal{F}_{u,v} = \{f(u,v)\}$, $\mathcal{F}_w = \{f(w)\}$:

$$H(s+d,\mathcal{F}) = H(s+d,\mathcal{F}_{u,v}) \times H(s+d,\mathcal{F}_w)$$

Usually (at least locally):

$$H(s+d,\mathcal{F}_{u,v}) \simeq \mathbb{K} \;(= \mathbb{R} \text{ or } \mathbb{C}) \;\Rightarrow\; H(s+d,\mathcal{F}) \simeq H(s+d,\mathcal{F}_w)$$

Typically:

$$\{u^\ell\} = \{A_\mu, \phi^\star, \dots\},$$

$$\{v^\ell\} = \{\partial_\mu C + \cdots, \frac{\delta \mathcal{L}}{\delta \phi} + \cdots, \dots\}$$

$$\{w^I\} = \{\tilde{T}^A, \tilde{C}^N\} \text{ with } \tilde{T}^A = T^A + \cdots,\; \tilde{C}^N = C^N + \cdots$$

with

$$(s+d)\,A_\mu = \partial_\mu C + \cdots\;, \quad (s+d)\,\phi^\star = \frac{\delta \mathcal{L}}{\delta \phi} + \cdots$$

$$(s+d)\,\tilde{T}^A = \tilde{C}^N \Delta_N \tilde{T}^A, \quad (s+d)\,\tilde{C}^N = \pm\tfrac{1}{2}\tilde{C}^K \tilde{C}^L \mathcal{F}_{LK}{}^N(\tilde{T}),$$

$$[\Delta_K, \Delta_L]_\pm = \mathcal{F}_{KL}{}^N(\tilde{T})\Delta_N$$

Emergence of SAC in local BRST cohomology

SAC arises within the linearized problem $H(s^{(0)}, \mathcal{F}_w)$ corresponding to

$$s^{(0)}\,\tilde{T}^A = \tilde{C}^N \Delta_N^{(0)} \tilde{T}^A, \quad s^{(0)}\,\tilde{C}^N = \pm\tfrac{1}{2}\tilde{C}^K\tilde{C}^L f_{LK}{}^N,$$

$$[\Delta_K^{(0)}, \Delta_L^{(0)}]_\pm = f_{KL}{}^N \Delta_N^{(0)} \quad (f_{KL}{}^N = \text{constant})$$

- SAC always emerges in this way within the local BRST cohomological analysis of standard supersymmetric field theories,
  - both for global and local SUSY
  - whether or not the algebra of SUSY transformations closes off-shell and/or modulo (other) gauge transformations
- Existence proof for variables $\tilde{T}$: FB, Lett. Math. Phys. 55 (2001) 149 [arXiv:math-ph/0103006]
- If the antifields are not eliminated as members of trivial pairs, the SAC arises as a "weak cohomology" (cohomology on-shell)

SUSY ladder equations

Strategy to compute SAC:
Decomposition in c-degree N_c (= degree in the translation ghosts)

$$s_{\mathsf{susy}} = d_c + d_\xi + s_{\mathsf{gh}}$$

$$d_c = c^a P_a\,,\ d_\xi = \xi_i^{\underline{\alpha}} Q^i_{\underline{\alpha}}\,,\ s_{\mathsf{gh}} = -\tfrac{1}{2}\, M^{ij}\,(\Gamma^a C^{-1})_{\underline{\alpha}\underline{\beta}}\,\xi_i^{\underline{\alpha}}\xi_j^{\underline{\beta}}\,\frac{\partial}{\partial c^a}$$

$$\omega = \sum_{p=m}^{M} \omega^p, \qquad N_c\,\omega^p = p\,\omega^p$$

SUSY ladder equations:

$$s_{\mathsf{susy}}\,\omega = 0 \ \Leftrightarrow \begin{cases} 0 = s_{\mathsf{gh}}\,\omega^m \\ 0 = d_\xi\,\omega^m + s_{\mathsf{gh}}\,\omega^{m+1} \\ 0 = d_c\,\omega^p + d_\xi\,\omega^{p+1} + s_{\mathsf{gh}}\,\omega^{p+2} \text{ for } m \le p \le M-2 \\ 0 = d_c\,\omega^{M-1} + d_\xi\,\omega^M \\ 0 = d_c\,\omega^M \end{cases}$$

Compute $H(s_{\mathrm{gh}})$ ("primitive elements" of SAC) and use the result to compute $H(s_{\mathrm{susy}})$ (spectral sequence technique)

Remark: analysis of the ladder eqs. is nontrivial only in c-degrees where $H(s_{\mathsf{gh}})$ does not vanish; typically (always?) these c-degrees are $\le D/2$.

Example in $D = 2$

SUSY algebra in $D = 2$ for Minkowski signature (-1,1),
$\Gamma^1 = -\mathrm{i}\,\sigma_1$, $\Gamma^2 = \sigma_2$, $C = \sigma_2$, $M^{ij} \equiv -\mathrm{i}\,\delta^{ij}$,
$(Q_{\underline{1}}, Q_{\underline{2}}) = (Q_+, Q_-)$ (two real Majorana-Weyl SUSYs):

$$(Q_+)^2 = -\tfrac{\mathrm{i}}{2}\,(P_1 + P_2),\ (Q_-)^2 = \tfrac{\mathrm{i}}{2}\,(P_1 - P_2),\ \{Q_+\,, Q_-\} = 0$$

Lagrangian for real boson φ and fermion $(\psi_{\underline{1}}, \psi_{\underline{2}}) = (\psi_+, \psi_-)$:

$$\begin{aligned} L &= -\tfrac{1}{2}\,\eta^{ab}\partial_a\varphi\,\partial_b\varphi - \mathrm{i}\,\psi^{\underline{\alpha}}(\Gamma^a C^{-1})_{\underline{\alpha\beta}}\,\partial_a\psi^{\underline{\beta}} \\ &= \tfrac{1}{2}\,(\partial_1\varphi)^2 - \tfrac{1}{2}\,(\partial_2\varphi)^2 + \mathrm{i}\,\psi_-(\partial_1 + \partial_2)\psi_- - \mathrm{i}\,\psi_+(\partial_1 - \partial_2)\psi_+ \end{aligned}$$

The action $\int dx^1 dx^2\, L$ is invariant under the **symmetry transformations**

$$\begin{aligned} &\delta_a\varphi = \partial_a\varphi\,,\ \ \delta_a\psi_{\underline{\alpha}} = \partial_a\psi_{\underline{\alpha}}\,,\ \ \delta_{\underline{\alpha}}\varphi = \psi_{\underline{\alpha}}\,,\ \ \delta_{\underline{\alpha}}\psi_{\underline{\beta}} = -\tfrac{\mathrm{i}}{2}\,(\Gamma^a C^{-1})_{\underline{\alpha\beta}}\,\partial_a\varphi \\ &\delta_\pm\varphi = \psi_\pm\,,\ \delta_+\psi_+ = -\tfrac{\mathrm{i}}{2}(\partial_1 + \partial_2)\varphi\,,\ \delta_-\psi_- = \tfrac{\mathrm{i}}{2}(\partial_1 - \partial_2)\varphi\,,\ \delta_\pm\psi_\mp = 0 \end{aligned}$$

SUSY algebra holds only on-shell, e.g.:

$$(\delta_+)^2\psi_- = 0 \approx -\tfrac{\mathrm{i}}{2}\,(\partial_1 + \partial_2)\psi_-$$

(Extended) BRST transformations for the example

$$
\begin{aligned}
s\,\varphi &= c^a\partial_a\varphi + \xi^+\psi_+ + \xi^-\psi_- \\
s\,\psi_+ &= c^a\partial_a\psi_+ - \tfrac{\mathrm{i}}{2}\,\xi^+\partial_+\varphi + \tfrac{1}{4}\,\xi^-\xi^-\psi^\star_+ - \tfrac{1}{4}\,\xi^+\xi^-\psi^\star_- \\
s\,\psi_- &= c^a\partial_a\psi_- + \tfrac{\mathrm{i}}{2}\,\xi^-\partial_-\varphi + \tfrac{1}{4}\,\xi^+\xi^+\psi^\star_- - \tfrac{1}{4}\,\xi^+\xi^-\psi^\star_+ \\
s\,\varphi^\star &= -\partial_+\partial_-\varphi + c^a\partial_a\varphi^\star - \tfrac{\mathrm{i}}{2}\,\xi^+\partial_+\psi^\star_+ + \tfrac{\mathrm{i}}{2}\,\xi^-\partial_-\psi^\star_- \\
s\,\psi^\star_+ &= 2\mathrm{i}\,\partial_-\psi_+ + c^a\partial_a\psi^\star_+ + \xi^+\varphi^\star \\
s\,\psi^\star_- &= -2\mathrm{i}\,\partial_+\psi_- + c^a\partial_a\psi^\star_- + \xi^-\varphi^\star \\
s\,c^+ &= \mathrm{i}\,\xi^+\xi^+ \\
s\,c^- &= -\mathrm{i}\,\xi^-\xi^- \\
s\,\xi^\pm &= 0
\end{aligned}
$$

where $\varphi^\star$, $\psi^\star_+$, $\psi^\star_-$ are the antifields to φ, ψ_+, ψ_- and

$$\partial_\pm = \partial_1 \pm \partial_2\,,\ c^\pm = c^1 \pm c^2,\ c^a\partial_a = c^1\partial_1 + c^2\partial_2 = \tfrac{1}{2}(c^+\partial_+ + c^-\partial_-)$$

On all fields and antifields:

$$s^2 = 0$$

Appropriate variables for computing $H(s)$

"Trivial pairs": BRST-doublets $\{u^\ell, v^\ell\}$ with $v^\ell = s\,u^\ell$:

$$\{u^\ell\} = \{\partial_+^m\partial_-^n\varphi^\star,\ \partial_+^m\partial_-^n\psi_+^\star,\ \partial_+^m\partial_-^n\psi_-^\star \,|\, m,n = 0,1,2,\dots\}$$
$$s\,\partial_+^m\partial_-^n\varphi^\star = -\partial_+^{m+1}\partial_-^{n+1}\varphi + \dots$$
$$s\,\partial_+^m\partial_-^n\psi_+^\star = 2\mathrm{i}\,\partial_+^m\partial_-^{n+1}\psi_+ + \dots$$
$$s\,\partial_+^m\partial_-^n\psi_-^\star = -2\mathrm{i}\,\partial_+^{m+1}\partial_-^n\psi_- + \dots$$

where $\partial_+^m = \partial_+\cdots\partial_+$ etc

"Nontrivial variables" $\{w^I\} = \{c^+,\, c^-,\, \xi^+,\, \xi^-,\, \tilde{T}^A\}$ with $s\,w^I = r^I(w)$:

$$\{\tilde{T}^A\} = \{\varphi_{(0,0)},\, \varphi_{(m+1,0)},\, \varphi_{(0,m+1)},\, \psi_{+(m,0)},\, \psi_{-(0,m)} \,|\, m = 0,1,2,\dots\}$$
$$\varphi_{(0,0)} = \varphi$$
$$\varphi_{(m+1,0)} = \partial_+^m(\partial_+\varphi - \tfrac{\mathrm{i}}{2}\,\xi^-\psi_-^\star + \tfrac{1}{2}\,c^-\varphi^\star)$$
$$\varphi_{(0,m+1)} = \partial_-^m(\partial_-\varphi + \tfrac{\mathrm{i}}{2}\,\xi^+\psi_+^\star - \tfrac{1}{2}\,c^+\varphi^\star)$$
$$\psi_{+(m,0)} = \partial_+^m(\psi_+ - \tfrac{\mathrm{i}}{4}\,c^-\psi_+^\star)$$
$$\psi_{-(0,m)} = \partial_-^m(\psi_- + \tfrac{\mathrm{i}}{4}\,c^+\psi_-^\star)$$

BRST transformations of the $\tilde{T}^A$ and SUSY algebra

$$s\,\tilde{T}^A = s_{\mathsf{susy}}\,\tilde{T}^A = (\tfrac{1}{2}\,c^+P_+ + \tfrac{1}{2}\,c^-P_- + \xi^+Q_+ + \xi^-Q_-)\,\tilde{T}^A$$

$\tilde{T}^A$	$\varphi_{(0,0)}$	$\varphi_{(m+1,0)}$	$\varphi_{(0,m+1)}$	$\psi_{+(m,0)}$	$\psi_{-(0,m)}$
$P_+\tilde{T}^A$	$\varphi_{(1,0)}$	$\varphi_{(m+2,0)}$	0	$\psi_{+(m+1,0)}$	0
$P_-\tilde{T}^A$	$\varphi_{(0,1)}$	0	$\varphi_{(0,m+2)}$	0	$\psi_{-(0,m+1)}$
$Q_+\tilde{T}^A$	$\psi_{+(0,0)}$	$\psi_{+(m+1,0)}$	0	$-\frac{\mathrm{i}}{2}\,\varphi_{(m+1,0)}$	0
$Q_-\tilde{T}^A$	$\psi_{-(0,0)}$	0	$\psi_{-(0,m+1)}$	0	$\frac{\mathrm{i}}{2}\,\varphi_{(0,m+1)}$

with $P_\pm = P_1 \pm P_2$. SUSY algebra:

$$[\,P_+\,,\,P_-\,] = [\,P_+\,,\,Q_+\,] = [\,P_+\,,\,Q_-\,] = [\,P_-\,,\,Q_+\,] = [\,P_-\,,\,Q_-\,] = 0,$$
$$(Q_+)^2 = -\tfrac{\mathrm{i}}{2}\,P_+\,,\quad (Q_-)^2 = \tfrac{\mathrm{i}}{2}\,P_-\,,\quad \{Q_+\,,\,Q_-\} = 0$$

Notice: P_+ and P_- map half of the generalized tensors $\varphi_{(m+1,0)}$, $\varphi_{(0,m+1)}$, $\psi_{+(m,0)}$, $\psi_{-(0,m)}$ to zero respectively and correspond to the action of ∂_+ and ∂_- on-shell (owing to $\partial_+(\partial_-)^{m+1}\varphi \approx 0$ etc)

Computation and result of $H(s)$

1. The trivial pairs drop from $H(s)$: $H(s) \simeq H(s_{\text{susy}})$
2. Computation of $H(s_{\text{gh}})$. Result:

$$s_{\text{gh}}\, f(c,\xi) = 0 \;\Leftrightarrow\; f(c,\xi) \sim a + \xi^+ a_+ + \xi^- a_- + \xi^+\xi^- a_{+-}\,;$$
$$a + \xi^+ a_+ + \xi^- a_- + \xi^+\xi^- a_{+-} \sim 0 \;\Leftrightarrow\; a = a_+ = a_- = a_{+-} = 0$$

3. Computation of $H(s_{\text{susy}})$ by analysis of the ladder equations:

cocycles :

$$s_{\text{gh}}\,\omega^m = 0 \;\Rightarrow\; m = 0,\; \omega^0 = a(\tilde T) + \xi^+ a_+(\tilde T) + \xi^- a_-(\tilde T) + \xi^+\xi^- a_{+-}(\tilde T)$$
$$d_\xi\,\omega^0 + s_{\text{gh}}\,\omega^1 = 0 \;\Rightarrow\; Q_+ a(\tilde T) = Q_- a(\tilde T) = 0,\; Q_- a_+(\tilde T) + Q_+ a_-(\tilde T) = 0$$
$$\Rightarrow \begin{cases} \text{gh} = 0: & \omega = a = \text{constant} \\ \text{gh} = 1: & \omega \sim (\xi^+ + \mathrm{i}\,c^+ Q_+) a_+(\tilde T) + (\xi^- - \mathrm{i}\,c^- Q_-) a_-(\tilde T) \\ \text{gh} = 2: & \omega \sim (\xi^+\xi^- + \mathrm{i}\,c^+\xi^- Q_+ - \mathrm{i}\,c^-\xi^+ Q_- - c^+c^- Q_+Q_-) a_{+-}(\tilde T) \\ \text{gh} > 2: & \omega \sim 0 \end{cases}$$

coboundaries :

$$\text{gh} = 1: \;\; \omega \sim 0 \;\Leftrightarrow\; a_+(\tilde T) = Q_+ b(\tilde T) \;\wedge\; a_-(\tilde T) = Q_- b(\tilde T)$$
$$\text{gh} = 2: \;\; \omega \sim 0 \;\Leftrightarrow\; a_{+-}(\tilde T) = Q_- b_+(\tilde T) + Q_+ b_-(\tilde T)$$

Sample solutions

Simple examples:

$$\mathsf{gh}=2:\ a_{+-}(\tilde{T})=f(\varphi)\ \Rightarrow$$
$$\omega=\xi^{+}\xi^{-}f(\varphi)+\mathrm{i}\,(c^{+}\xi^{-}\psi_{+(0,0)}-c^{-}\xi^{+}\psi_{-(0,0)})f'(\varphi)$$
$$-c^{+}c^{-}\psi_{+(0,0)}\psi_{-(0,0)}f''(\varphi)$$
$$\omega_2=-dx^{+}dx^{-}\Big[\psi_{+}\psi_{-}f''(\varphi)+\tfrac{1}{4}\,(\psi^{\star}_{-}\xi^{+}-\psi^{\star}_{+}\xi^{-})f'(\varphi)\Big]$$

$$\mathsf{gh}=1:\ a_{+}(\tilde{T})=\psi_{+(0,0)},\ a_{-}(\tilde{T})=-\psi_{-(0,0)}\ \Rightarrow$$
$$\omega=\xi^{+}\psi_{+(0,0)}-\xi^{-}\psi_{-(0,0)}+\tfrac{1}{2}\,(c^{+}\varphi_{(1,0)}-c^{-}\varphi_{(0,1)})$$
$$\omega_1=\tfrac{1}{2}\,(dx^{+}\partial_{+}-dx^{-}\partial_{-})\varphi+\cdots=(dx^{1}\partial_{2}+dx^{2}\partial_{1})\varphi+\ldots$$
$$\omega_2=-\tfrac{1}{2}\,dx^{+}dx^{-}\varphi^{\star}=dx^{1}dx^{2}\varphi^{\star}$$

More complicated example:

$$a_{+-}(\tilde{T})=\varphi_{(1,0)}\psi_{+(0,0)}\varphi_{(0,1)}\psi_{-(0,0)}\ \Rightarrow$$
$$\omega_2=dx^{+}dx^{-}(\tfrac{\mathrm{i}}{2}\,\partial_{+}\varphi\partial_{+}\varphi+\psi_{+}\partial_{+}\psi_{+})(\psi_{-}\partial_{-}\psi_{-}-\tfrac{\mathrm{i}}{2}\,\partial_{-}\varphi\partial_{-}\varphi)+\ldots$$

Comparision to Lie algebra cohomology (LAC)

Semisimple Lie algebra:

$$[\,\delta_i\,,\,\delta_j\,] = f_{ij}{}^k\delta_k$$

BRST-type differential:

$$s_{\,\mathsf{Lie}} = C^i\delta_i + \tfrac{1}{2}C^jC^kf_{kj}{}^i\frac{\partial}{\partial C^i}$$

LAC:

$$s_{\,\mathsf{Lie}}\,\omega(C,T) = 0 \;\;\Leftrightarrow\; \omega(C,T) = s_{\,\mathsf{Lie}}\,\eta(C,T) + \sum_r f^r(C)g_r(T)$$

$$\text{with } s_{\,\mathsf{Lie}}\,f^r(C) = 0 \;\;\wedge\;\; s_{\,\mathsf{Lie}}\,g_r(T) = 0$$

i.e., the representatives of the LAC factorize in Cs and Ts.

In sharp contrast, the representatives of the SAC do *not* factorize in this way because (normally) there are no nontrivial s_{susy}-invariants $g(\tilde{T})$!

However, the $s_{\,\mathsf{Lie}}$-invariants $f(C)$ have counterparts in $H(s_{\mathsf{susy}})$ given by the representatives of $H(s_{\mathsf{gh}})$.

Brief summary

- SAC is a cornerstone of the local BRST cohomology in **any** standard supersymmetric field theory, both for global and local SUSY and whether or not the algebra of the symmetry transformations closes off-shell
- SAC involves particularly useful variables for local BRST cohomology
- The differential

$$s_{\mathrm{gh}} = -\tfrac{1}{2}\, M^{ij}\, (\Gamma^a C^{-1})_{\underline{\alpha}\underline{\beta}}\, \xi_i^{\underline{\alpha}} \xi_j^{\underline{\beta}}\, \frac{\partial}{\partial c^a}$$

  plays a distinguished part and has no counterpart in standard (non-supersymmetric) Yang-Mills or gravity theories
- The representation of the translational generators P_a on the $\tilde{T}$ differs substantially from usual partial or covariant derivatives as it corresponds to a representation of partial or covariant derivatives *on-shell*

Recent work on SAC:

FB, Supersymmetry algebra cohomology I: Definition and general structure, J. Math. Phys. 51 (2010) 122302 [arXiv:0911.2118]

FB, Supersymmetry algebra cohomology II: Primitive elements in 2 and 3 dimensions, J. Math. Phys. 51 (2010) 112303 [arXiv:1004.2978]

FB, Supersymmetry algebra cohomology III: Primitive elements in four and five dimensions, J. Math. Phys. 52 (2011) 052301 [arXiv:1005.2102]

M. Movshev, A. Schwarz, R. Xu, Homology of Lie algebra of supersymmetries, arXiv:1011.4731

M. Movshev, A. Schwarz, R. Xu, Homology of Lie algebra of supersymmetries and of super Poincare Lie algebra, Nucl. Phys. B 854 (2012) 483 [arXiv:1106.0335]