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# New supersymmetric gauge theories with 2-form gauge potentials

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- Interactions of p-form gauge potentials

- Overview of the new models

- Example (N=2 TT-multiplet)

- Comments (open problems)

Interactions of p-form gauge potentials

$p$-form gauge fields $A = (1/p!) dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p} A_{\mu_1 \ldots \mu_p}$

with gauge transformations

$$\delta^{(0)} A = d\omega \;\Leftrightarrow\; \delta^{(0)} A_{\mu_1 \ldots \mu_p} = p \partial_{[\mu_1} \omega_{\mu_2 \ldots \mu_p]}$$

- are frequently present in susy multiplets
- play important rôle in string and sugra theories
- have very restricted interactions:

Henneaux, Knaepen 1997-1999:
Classification of first order interaction vertices that can be added consistently to free Lagrangians $\sim \sum F_{\mu_0 \ldots \mu_p} F^{\mu_0 \cdots \mu_p}$ (mod. field redefinitions):

1. Standard interactions = vertices that are $\delta^{(0)}$-invariant off-shell mod. total derivatives:

- Vertices depending on $A$'s only via field strengths and their derivatives (e.g. Born-Infeld actions)

2. Vertices giving rise to deformations of $\delta^{(0)}$ = vertices that are $\delta^{(0)}$-invariant on-shell in the free theory mod. total derivatives:

- exterior products of one $A$, $F$'s and ${}^*F$'s:

$$A \wedge F \wedge \ldots \wedge F \wedge \underbrace{{}^*F \wedge \ldots \wedge {}^*F}_{\text{at least one } {}^*F} \tag{1}$$

- additional couplings involving 1-form gauge fields (e.g.: cubic YM vertex)

Cubic couplings (1) of 1-form and 2-form gauge fields in 4d:

$A_2 \wedge {}^*F_3 \wedge {}^*F_3$ ('Freedman-Townsend vertices')
$A_1 \wedge {}^*F_2 \wedge {}^*F_3$ ('Henneaux-Knaepen vertices')
$A_1 \wedge F_2 \wedge {}^*F_3$ ('Chapline-Manton vertices')

# Overview of the new models

2-form gauge fields: $B^i_{\mu\nu} = -B^i_{\nu\mu}$ $(i = 1, 2, \ldots)$
1-form gauge fields: $A^a_\mu$ $(a = 1, 2, \ldots)$

Special interaction vertices ($H^{i\mu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\partial_\nu B^i_{\rho\sigma}$):

| | | |
|---|---|---|
| FT: | $f_{ijk}\, H^i_\mu H^j_\nu B^k_{\rho\sigma}\, \varepsilon^{\mu\nu\rho\sigma}$ | ('Freedman-Townsend') |
| HK: | $T_{iab}\, H^i_\mu F^{a\mu\nu} A^b_\nu$ | ('Henneaux-Knaepen') |
| CM: | $S_{iab}\, H^i_\mu F^a_{\nu\rho} A^b_\sigma\, \varepsilon^{\mu\nu\rho\sigma}$ | ('Chapline-Manton') |

where

$$f_{ijl} f_{klm} + f_{jkl} f_{ilm} + f_{kil} f_{jlm} = 0$$

$$[T_i, T_j] = f_{ijk}\, T_k$$

$$S_i T_j - S_j T_i - f_{ijk}\, S_k + \text{transposed} = 0$$

Solutions to third Eq. are, e.g., $S_i = N T_i$ (i.e. $S_{iab} = N_{ac} T_{icb}$), with arbitrary matrix $N$

New susy models:

Susy multiplets with gauge fields $B_{\mu\nu}$ and/or $A_\mu$:
N=1: T (tensor, "linear"), V (vector)
N=2: VT (vector-tensor), TT (double tensor), V (vector)

| susy | multiplets | interactions | papers |
|---|---|---|---|
| N=2 | VT , V | HK,CM | Claus et al[1,2,7]<br>Dragon,Theis[8]<br>Dragon et al[10] |
| N=2 | VT , V | CM | Grimm et al[3]<br>Dragon et al[4]<br>Buchbinder et al[5]<br>Ivanov,Sokatchev[9] |
| N=2 | VT | CM | Dragon,Kuzenko[6] |
| N=1 | T , V | HK,FT,CM | Brandt,Theis[11] |
| N=2 | VT | HK | Theis[12] |
| N=2 | TT , V | HK,FT | Brandt[13] |


[1] Claus,de Wit,Faux,Kleijn,Siebelink,Termonia,hep-th/9512143
[2] Claus,de Wit,Faux,Termonia, hep-th/9612203
[3] Grimm,Hasler,Herrmann, hep-th/9706108
[4] Dragon,Kuzenko,Theis, hep-th/9706169
[5] Buchbinder,Hindawi,Ovrut, hep-th/9706216
[6] Dragon,Kuzenko, hep-th/9709088
[7] Claus,de Wit,Faux,Kleijn,Siebelink,Termonia,hep-th/9710212
[8] Dragon,Theis, hep-th/9711025, hep-th/9805199
[9] Ivanov,Sokatchev, hep-th/9711038
[10] Dragon,Ivanov,Kuzenko,Sokatchev,Theis, hep-th/9805152
[11] Brandt,Theis, hep-th/9811180
[12] Theis, hep-th/0005044
[13] Brandt, hep-th/0005086


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[7] Sugra (all other models have global susy)

Remarks:

Non-susy FT interactions known a long time
(Ogievetsky,Polubarinov 1966; Freedman,Townsend 1981).
First susy FT interactions 1989
(Clark,Lee,Love 1989)

(Susy) CM interactions known a long time
(Nicolai,Townsend 1981; Bergshoeff et al 1982;
Chapline,Manton 1983).
Prominent application:
Green-Schwarz anomaly cancellation
(Green,Schwarz 1984)

First HK–CM interactions 1995 (susy).
Particular application:
gauging of central charge of the VT multiplet
(Claus et al 1995)

First HK–FT interactions 1997 (non-susy)
(Henneaux,Knaepen 1997)

First FT–HK–CM interactions 1998 (susy)
(Brandt,Theis 1998)

Example (Brandt 2000)

N=2 susy multiplets ($i=1,2$; $a=1,2$):

| | bosons | Weyl-fermions |
|---|---|---|
| TT | $B^i_{\mu\nu}$ $a^i$ $(h^i_\mu)$ | $\chi$ $\psi$ |
| $\mathsf{V}^a$ | $A^a_\mu$ $\phi^a$ | $\lambda^{ai}$ |

$h^i_\mu$: auxiliary fields

Lagrangian:

$$L = \partial_\mu a^i \partial^\mu a^i + h^i_\mu h^{\mu i} + 2 h^i_\mu H^{\mu i} - \mathrm{i}\chi\partial\bar{\chi} - \mathrm{i}\psi\partial\bar{\psi}$$
$$-\tfrac{1}{4}\widehat{F}^a_{\mu\nu}\widehat{F}^{a\mu\nu} + \tfrac{1}{2}\widehat{D}_\mu\phi^a\widehat{D}^\mu\bar{\phi}^a - 2\mathrm{i}\lambda^{ia}\widehat{D}\bar{\lambda}^{ia}$$

where

$$\begin{aligned}
\widehat{F}^a_{\mu\nu} &= \widehat{D}_\mu A^a_\nu - \widehat{D}_\nu A^a_\mu = \partial_\mu A^a_\nu + g^i h^i_\mu \varepsilon^{ab} A^b_\nu - (\mu \leftrightarrow \nu) \\
\widehat{D}_\mu \phi^a &= \partial_\mu \phi^a + g^i h^i_\mu \varepsilon^{ab}\phi^b \\
\widehat{D}\bar{\lambda}^{ia} &= \sigma^\mu(\partial_\mu \bar{\lambda}^{ia} + g^i h^i_\mu \varepsilon^{ab}\bar{\lambda}^{ib})
\end{aligned}$$

Here, $g^i \in \mathbb{R}$ are arbitrary (coupling) constants.

Gauge transformations:

$$\begin{aligned}
\delta_{\text{gauge}} A^a_\mu &= \widehat{D}_\mu \omega^a = \partial_\mu \omega^a + g^i h^i_\mu \varepsilon^{ab}\omega^b \\
\delta_{\text{gauge}} B^i_{\mu\nu} &= \tfrac{1}{4} g^i \omega^a \varepsilon^{ab}\varepsilon_{\mu\nu\rho\sigma}\widehat{F}^{b\rho\sigma} + \partial_\mu \omega^i_\nu - \partial_\nu \omega^i_\mu \\
\delta_{\text{gauge}} &= 0 \quad \text{on other fields}
\end{aligned}$$

Susy transformations (with constant anticommuting Weyl-spinors $\xi^i$):

$$\begin{aligned}
\delta_{\mathsf{susy}}\, A_\mu^a &= \varepsilon^{ij}\xi^i\sigma_\mu\bar\lambda^{ja} - \xi^i\Gamma^i\varepsilon^{ab}A_\mu^b + \text{c.c.}\\
\delta_{\mathsf{susy}}\, \phi^a &= 2\,\xi^i\lambda^{ia} - (\xi^i\Gamma^i + \bar\xi^i\bar\Gamma^i)\varepsilon^{ab}\phi^b\\
\delta_{\mathsf{susy}}\, \lambda^{ia} &= \tfrac{\mathrm{i}}{2}\,(\varepsilon^{ij}\xi^j\sigma^{\mu\nu}\widehat F_{\mu\nu}^a - \bar\xi^i\bar\sigma^\mu\widehat D_\mu\phi^a)\\
&\quad -(\xi^j\Gamma^j + \bar\xi^j\bar\Gamma^j)\varepsilon^{ab}\lambda^{ib}\\
\delta_{\mathsf{susy}}\, B_{\mu\nu}^i &= -\varepsilon^{ij}\xi^j\sigma_{\mu\nu}\chi + \xi^i\sigma_{\mu\nu}\psi\\
&\quad +\mathrm{i}g^i\varepsilon^{ab}(\bar\phi^a\xi^j\sigma_{\mu\nu}\lambda^{jb} + \varepsilon^{jk}A_{[\mu}^a\xi^j\sigma_{\nu]}\bar\lambda^{kb})\\
&\quad +\text{c.c.}\\
\delta_{\mathsf{susy}}\, a^i &= \tfrac{1}{2}\,(\xi^i\chi - \varepsilon^{ij}\xi^j\psi) + \text{c.c.}\\
\delta_{\mathsf{susy}}\, \chi &= -\bar\xi^i\bar\sigma^\mu(\varepsilon^{ij}h_\mu^j + \mathrm{i}\,\partial_\mu a^i)\\
\delta_{\mathsf{susy}}\, \psi &= -\bar\xi^i\bar\sigma^\mu(h_\mu^i + \mathrm{i}\,\varepsilon^{ij}\partial_\mu a^j)\\
\delta_{\mathsf{susy}}\, h_\mu^i &= \tfrac{\mathrm{i}}{2}\,\partial_\mu(\xi^i\psi - \varepsilon^{ij}\xi^j\chi) + \text{c.c.}
\end{aligned}$$

where

$$\Gamma^i = \tfrac{\mathrm{i}}{2}\,g^j(\varepsilon^{ij}\chi + \delta^{ij}\psi)$$

Algebra of susy and gauge transformations:

$$\begin{aligned}
[\delta_{\mathsf{susy}}, \delta'_{\mathsf{susy}}] &\approx \delta_{\mathsf{translation}} + \delta_{\mathsf{gauge}}\\
[\delta_{\mathsf{susy}}, \delta_{\mathsf{gauge}}] &\approx \delta'_{\mathsf{gauge}} \qquad \text{typical!}^*\\
[\delta_{\mathsf{gauge}}, \delta'_{\mathsf{gauge}}] &\approx 0
\end{aligned}$$

where $\approx$ is equality on-shell.

$$\begin{aligned}
*:\qquad \omega^{a\prime} &= (\xi^i\Gamma^i + \bar\xi^i\bar\Gamma^i)\varepsilon^{ab}\omega^b\\
\omega_\mu^{i\prime} &= -\tfrac{\mathrm{i}}{2}\,g^i\varepsilon^{ab}\varepsilon^{jk}\omega^a(\xi^j\sigma_\mu\bar\lambda^{kb} - \lambda^{kb}\sigma_\mu\bar\xi^j)
\end{aligned}$$

Elimination of $h^i_\mu$:

$$\begin{aligned} L &= -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \partial_\mu a^i\partial^\mu a^i + \tfrac{1}{2}\partial_\mu\phi^a\partial^\mu\bar{\phi}^a \\ &\quad -\mathrm{i}\chi\partial\bar{\chi} - \mathrm{i}\psi\partial\bar{\psi} - 2\mathrm{i}\lambda^{ia}\partial\bar{\lambda}^{ia} \\ &\quad +2h^i_\mu\mathcal{H}^{\mu i} + h^i_\mu K^{\mu i,\nu j}h^j_\nu \end{aligned}$$

where

$$\mathcal{H}^{\mu i} = H^{\mu i} - g^i\varepsilon^{ab}(\tfrac{1}{2}F^{a\mu\nu}A^b_\nu + \tfrac{1}{4}\phi^a \overleftrightarrow{\partial^\mu} \bar{\phi}^b + \mathrm{i}\lambda^{ja}\sigma^\mu\bar{\lambda}^{jb})$$

$$K^{\mu i,\nu j} = \eta^{\mu\nu}\delta^{ij} + \tfrac{1}{2}g^i g^j[\eta^{\mu\nu}(\phi^a\bar{\phi}^a - A^a_\rho A^{a\rho}) + A^{a\mu}A^{a\nu}]$$

Field equations for $h^i_\mu$ give $\boxed{h^i_\mu = -(K^{-1})_{\mu i,\nu j}\mathcal{H}^{\nu j}}$, $(K^{-1})_{\mu i,\rho k}K^{\rho k,\nu j} = \delta^\nu_\mu\delta^j_i$.

Substituting $(K^{-1})_{\mu i,\nu j}\mathcal{H}^{\nu j}$ for $h^i_\mu$ gives nonpolynomial action, gauge and susy transformations.

$$\begin{aligned} L &= -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \partial_\mu a^i\partial^\mu a^i + \tfrac{1}{2}\partial_\mu\phi^a\partial^\mu\bar{\phi}^a \\ &\quad -\mathrm{i}\chi\partial\bar{\chi} - \mathrm{i}\psi\partial\bar{\psi} - 2\mathrm{i}\lambda^{ia}\partial\bar{\lambda}^{ia} \\ &\quad -\mathcal{H}^{\mu i}\,(K^{-1})_{\mu i,\nu j}\,\mathcal{H}^{\nu j} \\ &= L_{\text{free}} + \underbrace{g^i\varepsilon^{ab}H^i_\mu F^{a\mu\nu}A^b_\nu}_{\text{HK vertex}} \\ &\quad + \underbrace{g^i\varepsilon^{ab}H^i_\mu(\tfrac{1}{2}\phi^a \overleftrightarrow{\partial^\mu} \bar{\phi}^b + 2\mathrm{i}\lambda^{ja}\sigma^\mu\bar{\lambda}^{jb})}_{\substack{\text{susy completion} \\ \text{of HK vertex}}} + O(g^i g^j) \end{aligned}$$

Nonpolynomial structure of this type arises necessarily in presence of HK or FT interaction vertices

## Comments (open problems)

- rôle of matter fields (scalar fields, fermions) in underlying geometry unclear
  (nonlinear representations?, . . . )

- systematic classification of susy interactions with $p$-form gauge fields missing
  (exception: lowest dimensional interactions of VTT-multiplet with itself and with hypermultiplets and V-multiplets[13])

- locally susy extension of models unknown
  (exception: specific sugra-models with VT-multiplets[7])

- generalization to higher dimensions (and higher $p$) yet unknown
  (no higher dimensional susy models with HK and/or FT interactions yet)